

Федеральное государственное автономное образовательное

учреждение высшего образования

«Национальный исследовательский университет ИТМО»

Факультет программной инженерии и компьютерной техники

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Дисциплина «Дискретная математика»

**Отчёт по домашней работе №6**

Вариант №68

Выполнил

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Санкт – Петербург 2024

**Сложение чисел с плавающей точкой**

|  |  |
| --- | --- |
| **A** | **B** |
| **0,342** | **0,299** |

**1. Формат Ф1.**

A = (0,342)10 = (0,578D5)16 = (0,578D5)16 · 160

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |

B = (0,299)10 = (0,4C8B44)16 = (0,4C8B44)16 · 160

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| XA | = | – | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| XB | = | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| (XA-XB)пр. | = |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

(XA-XB) = 0; XC = XA = XB = 0

а) Оба операнда положительные (A > 0, B > 0)

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| MA | = | + |  | . | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| MB | = |  | . | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 |
| MC | = |  |  | . | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 |

Результат нормализован.  
MC = . 1 0 1 0 0 1 0 0 0 0 1 0

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |

С\* = МС · 16Рс = (0,A42)16 · 160 = 0,64111.  
Определим абсолютную и относительную погрешности результата:  
ΔС = 0,641 – 0,64111 = -0,00011, где ΔС – абсолютная погрешность

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| δС = |  | -0,00011 |  | · 100% = 0,01716% |
| 0,641 |

где δС – относительная погрешность  
Результат получился с избытком.

б) A < 0, B > 0

Сложение мантисс будем проводить их прямым вычитанием. В качестве уменьшаемого используем мантиссу положительного операнда (B).

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| MA | = | – |  | . | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| MB | = |  | . | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 |
| MC | = |  |  | . | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |

Результат вычитания денормализован вправо.  
MC = . 1 0 1 1 0 0 0 0 0 0 0 0  
Т.к. выполнен сдвиг мантиссы влево, характеристику результата нужно уменьшить на 1 (ХC = ХC - 1 = -1).

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |

С\* = МС · 16Рс = (0,B00)16 · 16-1 = 0,04297.  
Определим абсолютную и относительную погрешности результата:  
ΔС = 0,043 – 0,04297 = 0,00003, где ΔС – абсолютная погрешность

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| δС = |  | 0,00003 |  | · 100% = 0,06977% |
| 0,043 |

где δС – относительная погрешность

Результат получен с недостатком.

c) A < 0, B > 0

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| MB | = | – |  | . | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 |
| MA | = |  | . | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| MC | = |  |  | . | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |

Результат вычитания денормализован вправо и представлен в дополнительном коде.  
MC = . 0 1 0 1 0 0 0 0 0 0 0 0  
Т.к. выполнен сдвиг мантиссы влево, характеристику результата нужно уменьшить на 1 (ХC = ХC - 1 = -1).

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |

С\* = МС · 16Рс = (-0,B00)16 · 16-1 = -0,04297.  
Определим абсолютную и относительную погрешности результата:  
ΔС = -0,043 – (-0,04297) = -0,00003, где ΔС – абсолютная погрешность

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| δС = |  | -0,00003 |  | · 100% = 0,06977% |
| -0,043 |

где δС – относительная погрешность

Результат получился с избытком.

**2. Формат Ф2.**

A = (0,342)10 = (0,578D5)16 = (0,101011110001101)2 · 2-1

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 0 |
| 19 | 18 | 17 | 16 | 15 | 14 | 13 | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |

B = (0,299)10 = (0,4C8B44)16 = (0,100110010001011)2 · 2-1

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| 19 | 18 | 17 | 16 | 15 | 14 | 13 | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| XA | = | – | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| XB | = | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| (XA-XB)доп. | = |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

(XA-XB) = 0; XC = XB = -1

а) Оба операнда положительные (A > 0, B > 0):

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| MA | = | + |  | . | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 0 |
| MB | = |  | . | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| MC | = |  | 1 | . | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |

Результат сложения денормализован влево.  
MC = . 1 0 1 0 0 1 0 0 0 0 0 1  
Т.к. выполнен сдвиг мантиссы вправо, характеристику результата нужно увеличить на 1 (ХC = ХC + 1 = 0).

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| 19 | 18 | 17 | 16 | 15 | 14 | 13 | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |

С\* = МС · 2Рс = (0,101001000001)2 · 20 = 0,64087.  
Определим абсолютную и относительную погрешности результата:  
ΔС = 0,641 – 0,64087 = 0,00013, где ΔС – абсолютная погрешность

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| δС = |  | 0,00013 |  | · 100% = 0,02028% |
| 0,641 |

где δС – относительная погрешность

Результат получился с недостатком.

б) A > 0, B < 0

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| MA | = | – |  | . | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 0 |
| MB | = |  | . | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| MC | = |  |  | . | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 |

Результат вычитания денормализован вправо.  
MC = . 1 0 1 1 0 0 0 0 1 0 0 0  
Т.к. выполнен сдвиг мантиссы влево, характеристику результата нужно уменьшить на 3 (ХC = ХC - 3 = -4).

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 19 | 18 | 17 | 16 | 15 | 14 | 13 | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |

С\* = МС · 2Рс = (0,101100001)2 · 2-4 = 0,04309.  
Определим абсолютную и относительную погрешности результата:  
ΔС = 0,043 – 0,04309 = -0,00009, где ΔС – абсолютная погрешность

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| δС = |  | -0,00009 |  | · 100% = 0,2093% |
| 0,043 |

, где δС – относительная погрешность  
Результат получился с избытком.

c) A < 0, B > 0

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| MB | = | – |  | . | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| MA | = |  | . | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 0 |
| MC | = |  |  | . | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |

Результат вычитания денормализован вправо и представлен в дополнительном коде.  
MC = . 0 1 0 0 1 1 1 1 1 0 0 0  
Т.к. выполнен сдвиг мантиссы влево, характеристику результата нужно уменьшить на 3 (ХC = ХC - 3 = -4).

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 19 | 18 | 17 | 16 | 15 | 14 | 13 | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |

С\* = МС · 2Рс = (-0,101100001)2 · 2-4 = -0,04309.  
Определим абсолютную и относительную погрешности результата:  
ΔС = -0,043 – (-0,04309) = 0,00009, где ΔС – абсолютная погрешность

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| δС = |  | 0,00009 |  | · 100% = 0,2093% |
| -0,043 |

где δС – относительная погрешность

Результат получен с недостатком.

Причины возникновения погрешности:

1. Неточное представление операндов.
2. Потеря значащих разрядов мантиссы одного из операндов при уравнивании порядков.
3. Потеря значащих разрядов мантиссы результата при его нормализации сдвигом мантиссы вправо.

В формате Ф2 результаты точнее из-за того, что операнды представлены точнее и при нормализации результата сдвиг производился на один двоичный разряд, а не на четыре.